The Sky's the Limit (Unless You're the CN Tower)

1 Introduction

The animated movie *Up* showed us what's possible when you combine a grumpy old man, a talking dog, and a truly staggering amount of balloons. But let's be honest, we all had the same thought: That's not how physics works. This essay is dedicated to the people who watched that movie and immediately started doing the math in their heads. We're going to take a problem that's straight out of a children's film and solve it using calculus and physics. The target? The CN Tower. The solution? A whole lot of balloons. To solve this problem, we will proceed by dividing it into two main parts. First we will apply calculus to determine the volume and surface area of a single balloon. Then, we will use that to calculate the balloon's buoyant force and scale that number to match the CN Tower's immense mass in order to find the total number of balloons required.

The balloons that will be used to lift the CN Tower are represented by the following 2D piecewise function that resembles the half-section of a balloon.

$$f(x) = \begin{cases} x, & 0 \leq x \leq 10, \\ \sqrt{200-(x-20)^2}, & 10 \leq x \leq 20+10\sqrt{2} \end{cases} \tag{1}$$

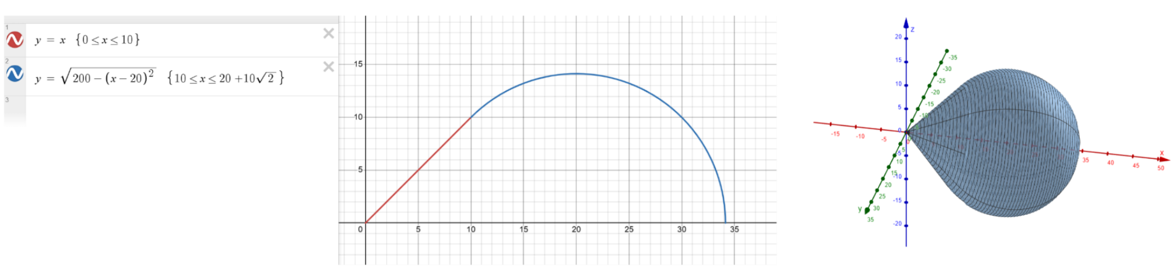

Figure 1: Balloon Half-section (left)[1] and 3-D Visualization (right)[2].

Note: The balloon's shape is approximated using a symmetrical, piecewise-defined model, though it doesn't perfectly reflect the true geometry of commercial latex balloons.

2 Mathematical Modeling of the Balloon

2.1 Surface Area Calculation via Integration

To accurately model our balloon, we first need to determine its volume and surface area. We will use the disk method to calculate the volume of the solid of revolution and the surface area of revolution formula to calculate its surface area. This involves integrating the piecewise function that defines the balloon's half-section.

First, let's find the surface area of the balloon by integrating each piece of the function. The general formula[3] for the surface area of a solid of revolution around the x-axis is:

$$\text{S} = \int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx \tag{2}$$

For the surface area of the first piecewise function, where $f(x) = x$, its derivative is $f'(x) = 1$. The calculation is as follows:

$$\begin{aligned}
S_1 &= \int_0^{10} 2\pi x\sqrt{1+(1)^2}\,dx \quad \text{(Substitute function and derivative)}\\
&= 2\pi\sqrt{2}\int_0^{10} x\,dx \quad \text{(Simplify and Constant Rule)}\\
&= 2\pi\sqrt{2}\left[\frac{x^2}{2}\right]_0^{10} \quad \text{(Power rule of integration)}\\
&= 2\pi\sqrt{2}\left(\frac{10^2}{2}-\frac{0^2}{2}\right) \quad \text{(Evaluate at limits of integration)}\\
&= 100\pi\sqrt{2} \approx 444.29\ \text{cm}^2
\end{aligned} \tag{3}$$

Next, we calculate the surface area for the second piecewise function, $f(x) = \sqrt{200-(x-20)^2}$. To do this, we first find its derivative and simplify the term $\sqrt{1+(f'(x))^2}$.

$$f'(x) = \frac{-(x-20)}{\sqrt{200-(x-20)^2}} \tag{4}$$

We calculate the term $\sqrt{1+(f'(x))^2}$:

$$
\begin{aligned}
\sqrt{1+(f'(x))^2} &= \sqrt{1+\left(\frac{-(x-20)}{\sqrt{200-(x-20)^2}}\right)^2} \\
&= \sqrt{1+\frac{(x-20)^2}{200-(x-20)^2}} \\
&= \sqrt{\frac{200-(x-20)^2+(x-20)^2}{200-(x-20)^2}} \quad \text{(Finding common denominator)} \\
&= \frac{\sqrt{200}}{\sqrt{200-(x-20)^2}}
\end{aligned} \tag{5}
$$

Since $f(x) = \sqrt{200-(x-20)^2}$, we can simplify and say $\sqrt{1+(f'(x))^2} = \frac{\sqrt{200}}{f(x)}$

Now we can solve the integral for the surface area of the second piece:

$$
\begin{aligned}
S_2 &= \int_{10}^{20+10\sqrt{2}} 2\pi\, f(x) \left(\frac{\sqrt{200}}{f(x)}\right) dx \\
&= 2\pi\sqrt{200} \int_{10}^{20+10\sqrt{2}} 1\, dx \quad \text{(f(x) terms cancel out)} \\
&= 2\pi\sqrt{200}\, [\, x \,]_{10}^{20+10\sqrt{2}} \quad \text{(Power rule of integration)} \\
&= 2\pi\sqrt{200} \left((20+10\sqrt{2})-10\right) \quad \text{(Evaluate at limits of integration)} \\
&= 2\pi(10\sqrt{2})(10+10\sqrt{2}) \quad \text{(Substitute } \sqrt{200}=10\sqrt{2}\text{)} \\
&= 200\pi\sqrt{2}+400\pi \approx 2145.21 \text{ cm}^2
\end{aligned} \tag{6}
$$

The **total surface area** of our balloon is the sum of these two parts:

$$
\begin{aligned}
S_{\text{Total}} &= S_1 + S_2 \\
&\approx 444.29 + 2145.21 \approx 2589.5 \text{ cm}^2
\end{aligned} \tag{7}
$$

2.2 Volume Calculation via Integration

Now, we calculate the **volume of the balloon** using the disk method. The general formula is $V = \int_a^b \pi\, [f(x)]^2\, dx$. We apply this to each function in our piecewise model. For the volume of the first piece, where $f(x) = x$ on the interval [0,10]:

$$
\begin{aligned}
V_1 &= \int_0^{10} \pi x^2 \, dx \\
&= \pi \left[\frac{x^3}{3} \right]_0^{10} \quad \text{(Power rule)} \\
&= \pi \left(\frac{10^3}{3} - \frac{0^3}{3} \right) \quad \text{(Evaluate at limits of integration)} \\
&= \frac{1000\pi}{3} \approx 1047.2 \text{ cm}^3
\end{aligned}
\tag{8}
$$

For the volume of the second piece, where $f(x) = \sqrt{200 - (x-20)^2}$ on the interval $[10,\ 20 + 10\sqrt{2}]$:

$$
\begin{aligned}
V_2 &= \int_{10}^{20+10\sqrt{2}} \pi \left(\sqrt{200 - (x-20)^2} \right)^2 dx \\
V_2 &= \pi \int_{10}^{20+10\sqrt{2}} \left[200 - (x-20)^2 \right] dx \quad \text{(Constant rule)}
\end{aligned}
\tag{9}
$$

Let $u = x - 20$, so $du = dx$.

The new limits of integration are: When $x = 10, u = -10$ and when $x = 20 + 10\sqrt{2}, u = 10\sqrt{2}$.

$$
\begin{aligned}
V_2 &= \pi \int_{-10}^{10\sqrt{2}} (200 - u^2) du \\
V_2 &= \pi \left[200u - \frac{u^3}{3} \right]_{-10}^{10\sqrt{2}} \\
V_2 &= \pi \left[200(10\sqrt{2}) - \frac{(10\sqrt{2})^3}{3} - \left(200(-10) - \frac{(-10)^3}{3} \right) \right] \\
&\approx 11159.83 \text{ cm}^3 \quad \text{(Using Calculator[1])}
\end{aligned}
\tag{10}
$$

The total volume is the sum of the two parts. Therefore,

$$
\begin{aligned}
V_{\text{Total}} &\approx 1047.2 + 11159.829 \\
&\approx 12207.029 \text{ cm}^3 \approx 0.012207029 \text{ m}^3
\end{aligned}
\tag{11}
$$

3 Upward Force and No. of Balloons Required to lift CN tower

The net upward force exerted by a single helium balloon is determined by Archimedes' principle.

It states that the buoyant force, which pushes the balloon upward, is equal to the weight of the air it displaces. The downward forces are the weight of the helium inside the balloon and the weight of the latex material itself. We must subtract these downward forces from the upward buoyant force to find the net lift. Before doing any calculations it's important to note that we assume static lift at sea level (ignoring wind and altitude effects) and that each balloon is made from uniform latex material.

First, we use the surface area and an assumed thickness to find the mass of the balloon material itself. The soft latex balloon is assumed to have a thickness of 0.0000125 m and a density of 1100 kg/m^3 [4]. Note that the following calculations have been done using a calculator.[1].

$$\begin{aligned} V_{\text{rubber}} &= \text{Surface Area} \times \text{Thickness} \\ &= 0.25895 \text{ m}^2 \times 0.0000125 \text{ m} = 3.236875 \times 10^{-6} \text{ m}^3 \end{aligned} \tag{12}$$

$$\begin{aligned} m_{\text{balloon}} &= V_{\text{rubber}} \times \rho_{\text{rubber}} \\ &= (3.2369 \times 10^{-6} \text{ m}^3)(1100 \text{ kg/m}^3) \approx 0.00356 \text{ kg} = 3.56 \text{ g} \end{aligned} \tag{13}$$

Now we calculate the individual forces. First, we find the **buoyant force**. We use the density of air at STP, which is approximately 1.225 kg/m^3 [5]:

$$\begin{aligned} F_{\text{buoyant}} &= (\rho_{\text{air}} \times V_{\text{balloon}}) \times g \\ &= (1.225 \text{ kg/m}^3 \times 0.012207029 \text{ m}^3) \times 9.81 \text{ m/s}^2 \\ &\approx 0.1465 \text{ N} \end{aligned} \tag{14}$$

Next, we compute the downward force exerted by the helium inside the balloon. We use the density of helium at STP, which is 0.1785 kg/m^3[6]:

$$\begin{aligned} F_{\text{He}} &= (\rho_{\text{He}} \times V_{\text{balloon}}) \times g \\ &= (0.1785 \text{ kg/m}^3 \times 0.012207029 \text{ m}^3) \times 9.81 \text{ m/s}^2 \\ &\approx 0.02136 \text{ N} \end{aligned} \tag{15}$$

Finally, we figure out the downward force of the latex balloon itself. To do so, the mass of the balloon material is used, which is calculated to be 3.56 g or 0.00356 kg.

$$\begin{aligned} F_{\text{latex}} &= m_{\text{balloon}} \times g \\ &= 0.00356 \text{ kg} \times 9.81 \text{ m/s}^2 \approx 0.0349 \text{ N} \end{aligned} \tag{16}$$

By subtracting the downward forces from the buoyant force, we find the **net upward force of a single balloon:**

$$\begin{aligned} F_{\text{net}} &= F_{\text{buoyant}} - F_{\text{He}} - F_{\text{latex}} \\ &= 0.1465 \text{ N} - 0.02136 \text{ N} - 0.0349 \text{ N} \\ &\approx 0.09044 \text{ N} \end{aligned} \tag{17}$$

The **net upward force** provided by a single balloon is approximately **0.09044 Newtons**.

Total Count of Balloons

To determine the number of balloons needed, we must first find the **total downward force** exerted by the CN Tower. The mass of the tower is approximately $117,910,000$ kg [7].

$$\begin{aligned} F_{\text{Tower}} = M_{\text{Tower}} \times g &= 117,910,000 \text{ kg} \times 9.81 \text{ m/s}^2 \\ &\approx 1,156,697,100 \text{ N} \end{aligned} \tag{18}$$

Finally, to find the total number of balloons required to lift the CN Tower, we divide the tower's downward force by the net upward force of a single balloon, which we determined previously.

$$\begin{aligned} N_{\text{balloons}} &= \frac{F_{\text{Tower}}}{F_{\text{net}}} \\ &= \frac{1,156,697,100 \text{ N}}{0.09044 \text{ N/balloon}} \approx 12,789,662,760 \end{aligned} \tag{19}$$

We would need approximately **12.8 billion balloons** to lift the CN Tower. It is also essential to note that the model neglects the mass of cords, harnesses, and attachment systems.

4 Conclusion

While the calculations provide a precise numerical answer, they rely on several simplifying assumptions to reach this result. One major challenge in lifting the CN Tower is its structural design. It is engineered to withstand immense compressive forces and resist lateral wind loads. Its foundation transfers the tower's massive weight deep into the earth. An upward pull could cause the structure to separate into sections before leaving the ground.

Another limitation is our assumption of constant environmental conditions such as standard temperature and pressure. In reality, the buoyant force would change with variations in air density, temperature, and pressure at different altitudes. So, once the tower rises, air becomes thinner and buoyant force weakens. Even slight density changes would stop the tower from climbing. The immense collective surface area of billions of balloons would also be highly vulnerable to wind forces. The balloons would not provide a stable and consistent lifting force resulting in destabilizing the lift.

The model also assumes that the balloons can be arranged in a perfect way with no empty space between them and without pressing against each other. In this ideal arrangement, every balloon would keep its full round shape and produce the same amount of lift. In reality, round balloons cannot completely fill space without leaving gaps, and when packed tightly they deform and lose some volume, which would reduce their lifting power and require more balloons than the calculated number. **Considering these limitations and assumptions, it has been determined that approximately 12.8 billion balloons would be required to lift the CN Tower at sea level under ideal conditions.** To reach to this conclusion, we first modeled the volume of a single balloon with calculus, then used that to calculate the buoyant force before scaling the result to the tower's mass.

References

[1] "Graphing calculator," Desmos, https://www.desmos.com/calculator (accessed Aug. 5, 2025).

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[3] G. Strang and E. "Jed" Herman, "Preface - calculus volume 1," OpenStax, https://openstax.org/books/calculus-volume-1/pages/preface (accessed Aug. 10, 2025).

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[5] L. Hoyos, "Air Density calculator," What is Air's Density?, https://www.calctool.org/atmospheric-thermodynamics/air-density (accessed Aug. 8, 2025).

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[7] "CN Tower: Destination toronto," Tourism Toronto, https://www.destinationtoronto.com/things-to-do/attractions/must-see-attractions/cn-tower/#:~:text=The%20360%20Restaurant%20takes%2072,structure%20in%20the%20western%20hemisphere (accessed Aug. 14, 2025).